Saving to Overcome Borrowing Constraints: Implications for Small Business Entry and Exit

Simon C. Parker

ABSTRACT. This paper shows how a dynamic analysis of entrepreneurial choice, in which inter-temporal savings plans can be adjusted to build up capital, allows agents to overcome borrowing constraints. This casts fresh doubt on the notion that borrowing constraints deter entrepreneurial entry in the long run. Impatient agents are shown to voluntarily remain constrained indefinitely, whereas patient agents become entrepreneurs. The model also explains temporary and permanent closures of viable businesses. The model's predictions are consistent with evidence of greater savings behaviour by entrepreneurs.

1. Introduction

This paper analyses the decision to become and remain an entrepreneur in a dynamic setting where borrowing constraints exist. Unlike Evans and Jovanovic (1989), who describe the negative impact of exogenous borrowing constraints on entrepreneurship in a single-period static model, I show how dynamic modelling allows agents to overcome these constraints by adjusting their savings behaviour. In particular, entrepreneurs have an incentive to save more than employees – a prediction that is borne out by recent U.S. evidence (Quadrini, 1999). This story contrasts with the prevailing view that individuals are passive victims of borrowing constraints, and adds further to growing scepticism in the literature about their practical importance.[1] I also show how some, relatively impatient, individuals may voluntarily and optimally succumb to constraints that prevent them from setting up or remaining in viable businesses, even though this would make them financially better off (I term this "voluntary constrainedness"). Also, the conjunction of borrowing constraints with a dynamic inter-temporal analysis can account for temporary and permanent closures of viable businesses as outcomes of optimising behaviour. Another key result is that individuals with a low subjective discount rate are more likely to set up in business, because for them, the opportunity cost of the required capital accumulation is lowest.

The paper is organised as follows. Section 2 explains the assumptions and structure of the life-cycle model. Section 3 states the solution and several key results. The robustness of the model to the relaxation of various assumptions is examined in Section 4. Section 5 concludes.

2. The model: assumptions and structure

In this paper I consider the case where individuals are given access to credit to start a new business, but are limited in the amount they can borrow.[2] Suppose banks impose a constant upper bound on agents' debt, $-\underline{a} \geq 0$. For simplicity, I start by deriving results for the special case $\underline{a} = 0$ (i.e., no debt allowed); it will be shown later that relaxing this assumption to permit positive time-variable debt and collateral (secured lending) leaves the key results unchanged.

Agents live over the time interval $t \in [0, T]$, where the terminal time T and all events within the interval are certain. Initially, agents are assumed to have no bequest motive. These assumptions are relaxed later. Agents supply one unit of labour inelastically; they derive instantaneous utility from consumption, $u[c(t)]$, with

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University of Durham
Department of Economics and Finance
23–26 Old Elvet
Durham DH1 3HY
U.K.
E-mail: s.c.parker@durham.ac.uk

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$u_c > 0$, $u_{cc} < 0$, and the Inada conditions $u_c(\infty) = 0$, $u_c(0) = \infty$. If agents choose paid employment, they receive a return of $w > 0$ per period, assumed constant for simplicity. If they choose entrepreneurship, they receive $f[k(t)]$, where $k(t) \geq 0 \ \forall t$ is physical capital, and $f[\cdot]$ is the production function, assumed to be concave ($f_k > 0$, $f_{kk} < 0$) and time-invariant.[3] In general, $f[\cdot]$ may also depend on some measure of entrepreneurial ability. For notational brevity, this will be implicit in the following, and only made explicit where necessary.

In each period, agents must choose how much to invest and consume, and also whether to work as employees or entrepreneurs. Given any $k(t)$, agents choose $\max\{w, f[k(t)]\}$; equivalently, the problem may be posed as

$$\max_{v} \ \{(f[k(t)])^{v} w^{1-v}\}, \qquad v = \{0, 1\}$$

where v is a binary "switch" variable. An agent is an entrepreneur if $v = 1$ and an employee if $v = 0$. Values of $k(t)$ are endogenous in the complete decision problem, for which the objective function is:

$$\max_{\{c(t),\, i(t),\, v(t)\}} \int_0^T u[c(t)]e^{-\rho t} \mathrm{d}t. \tag{1}$$

Here $\rho > 0$ is an agent's subjective intertemporal discount rate, and i denotes investment in physical capital. Physical and financial assets evolve according to the respective intertemporal budget constraints

$$\dot{k}(t) = i(t) - \delta k(t), \tag{2}$$

$$\dot{a}(t) = ra(t) + (f[k(t)])^{v} w^{1-v} - c(t) - i(t), \tag{3}$$

where $r > 0$ is the interest rate, assumed to be constant and identical for both borrowers and lenders; and $\delta \geq 0$ is the rate of depreciation of physical capital. Both state variables $k(t)$ and $a(t)$ are restricted to be non-negative for all $t \in [0, T]$; and $k(0) = k_0$, $a(0) = a_0$ (given).

The structure of this problem differs in several important respects from other "no-debt" problems studied by Artle and Varaiya (1978) and Schworm (1980). The former studied a life cycle consumption problem where income is exogenous; the latter studied a profit maximisation problem for an incorporated firm in which physical capital can be bought, but not sold. Neither model seems the ideal vehicle for studying dynamic entrepreneurial choice under borrowing constraints. Most entrepreneurs operate small businesses and have maximisation of utility rather than profits as their central objective. In this sense they have a life cycle problem closer to Artle and Varaiya. However, their income is not exogenous, as assumed by Artle and Varaiya, but endogenous, depending on their investment decisions, as modelled by Schworm. But unlike Schworm's assumption, entrepreneurs typically have considerable flexibility in assembling and dismantling physical capital, and are able to sell physical capital upon exit, often to competitors. All these "realistic" features are present in our model.

The maximisation of Eq. (1) subject to Eq. (2), Eq. (3), the initial conditions and non-negativity restrictions is an optimal control problem with two state space constraints. Suppressing the time index for simplicity, the Lagrangean is:

$$\mathcal{L} = u(c)e^{-\rho t} + (\Lambda_a + \Theta_a)\,(ra + (f(k))^{v} w^{1-v} - c - i) + (\Lambda_k + \Theta_k)\,(i - \delta k), \tag{4}$$

where Λ_a, Λ_k are co-state variables, and $\Theta_a \geq 0$, $\Theta_k \geq 0$ measure the tightness of the state constraints (see below). The Lagrangean is a linear function of the control variable i, a non-linear function of the control c, and involves a binary control v. The solution to this problem will therefore involve switches of v and "bang-bang" solutions for i.

3. The solution and results

3.1. *Solution and voluntary constrainedness*

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c} = u_c e^{-\rho t} - (\Lambda_a + \Theta_a) = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial i} = \Lambda_k + \Theta_k - (\Lambda_a + \Theta_a) = 0 \tag{6}$$

$$\max \mathcal{L} \quad \text{with respect to } v \tag{7}$$

$$\dot{k}(t) = i(t) - \delta k(t)$$

$$\dot{a}(t) = ra(t) + (f[k(t)])^{v} w^{1-v} - c(t) - i(t)$$

$$a \geq 0 \qquad \Theta_a a = 0 \qquad \forall t$$

$$\dot{\Theta}_a \leq 0 \quad [= 0 \text{ when } a > 0]$$

$$k \geq 0 \quad \Theta_k k = 0 \quad \forall t$$

$$\dot{\Theta}_k \leq 0 \quad [= 0 \text{ when } k > 0]$$

$$\dot{\Lambda}_a = -\frac{\partial \mathcal{L}}{\partial a} = -r(\Lambda_a + \Theta_a) \tag{8}$$

$$\dot{\Lambda}_k = -\frac{\partial \mathcal{L}}{\partial k} = -f_k(\Lambda_a + \Theta_a) + \delta(\Lambda_k + \Theta_k) \quad \text{if} \quad v = 1 \tag{9}$$

where a dot denotes a time derivative; Mangasarian's second order condition indicates these equations provide a maximum. From Eq. (7) we obtain

$$v^* = \begin{cases} 1 & \text{if } (\Lambda_a + \Theta_a)\,(f(k) - w) \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

It can be verified from Eq. (5) and Eq. (8) that $\Lambda_a > 0$ must hold for all t, in order to have positive marginal utility and hence positive consumption as required by the Inada conditions. Since also $\Theta_a \geq 0$, Eq. (10) implies that agents will be indifferent between employment and entrepreneurship when $k = m$, where $m > 0$ satisfies $f(m) = w$, and will choose entrepreneurship when they have at least m units of physical capital. Otherwise, they will liquidate any remaining physical capital into financial capital ($i^* = -k$) and choose paid employment, setting $i^* = 0$ in all periods in which they remain in this state.[4]

Denote two people with identical tastes and abilities by *B* and *N*, representing those facing binding and non-binding borrowing constraints in their financial assets, respectively. Assume for the present that both agents have $k_0 > m$, i.e., $v(0) = 1$, so both start as entrepreneurs. From the first order conditions, we have the following proposition:

Proposition 1: *While constrained, B has (i) lower consumption, and (ii) a lower capital stock, than N; and (iii) unlike N, B's investment flows are constrained by current income, being financed out of current savings.*

Proof. (i) From Eq. (5), $u_c = e^{\rho t}(\Lambda_a + \Theta_a)$. For N, $\Theta_a = 0$; for B, $\Theta_a > 0$. Hence the marginal utility for B exceeds that of N, which implies lower consumption for B than N. (ii) From Eq. (6), $\Lambda_k = \Lambda_a + \Theta_a - \Theta_k$. Differentiate with respect to t and equate with the RHS of Eq. (9), putting $\dot{\Theta}_k = \Theta_k = 0$ (since $k_0 > 0$) and $v = 1$:

$$-r(\Lambda_a + \Theta_a) + \dot{\Theta}_a = -f_k(\Lambda_a + \Theta_a) + \delta\Lambda_k.$$

For N, $a > 0$ so $\dot{\Theta}_a = \Theta_a = 0$ and $\Lambda_k = \Lambda_a$. Hence $f_k = r + \delta$, i.e., the marginal product of capital equals the marginal user cost. For B, $a = 0$ so $\dot{\Theta}_a < 0$ and

$$f_k = r + \delta - \frac{\dot{\Theta}_a}{\Lambda_a + \Theta_a}.$$

Since $\dot{\Theta}_a < 0$ and $\Lambda_a + \Theta_a > 0$ for B, the marginal product of B's capital exceeds that of N, i.e., B uses less capital than N. (iii) For B, $\dot{a} = a = 0$, so $i^* = f(k) - c^*$, i.e., all available savings are invested, where c^* solves Eq. (5). For N, the optimal capital stock, $\bar{k}$ say, solves $f_{\bar{k}} = r + \delta$: after this is achieved, N subsequently sets $i^* = 0$. □

Figure 1 illustrates N and B's optimal capital paths, for the case where B has positive savings, and assuming henceforth $w < f(\bar{k})$, where $\bar{k}$ is the optimal unconstrained capital stock (else no workers would choose entrepreneurship at all). Both paths are piecewise continuous, the former being vertical then horizontal ($k_0\,\overline{k}\overline{k}_{TC}$), the latter rising diagonally before becoming horizontal (i.e., $k_0\,\overline{k}_B\,\overline{k}_{TC}$). From the proof of Proposition 1, greater depreciation δ reduces $\bar{k}$ and hence the severity of the borrowing constraint given any initial assets $\langle a_0, k_0\rangle$. However, the greater is δ, the harder it is to obtain and maintain this $\bar{k}$.

Proposition 1 shows why borrowing constraints are important. In their absence, all agents will optimally borrow to maximise income with $k^* = \bar{k}$, becoming entrepreneurs and following an optimal unconstrained consumption path.[5] Proposition 1 also states that financially constrained entrepreneurs have a sub-optimal capital stock, which they must build up by investing all their available savings. To facilitate achievement of the optimal capital stock $\bar{k}$, they reduce consumption. This is consistent with evidence that entrepreneurs have higher savings rates than employees, and implies that agents have some control over the incidence and duration of their borrowing constraints.[6] However, as I now show, it may still be optimal for them to choose to

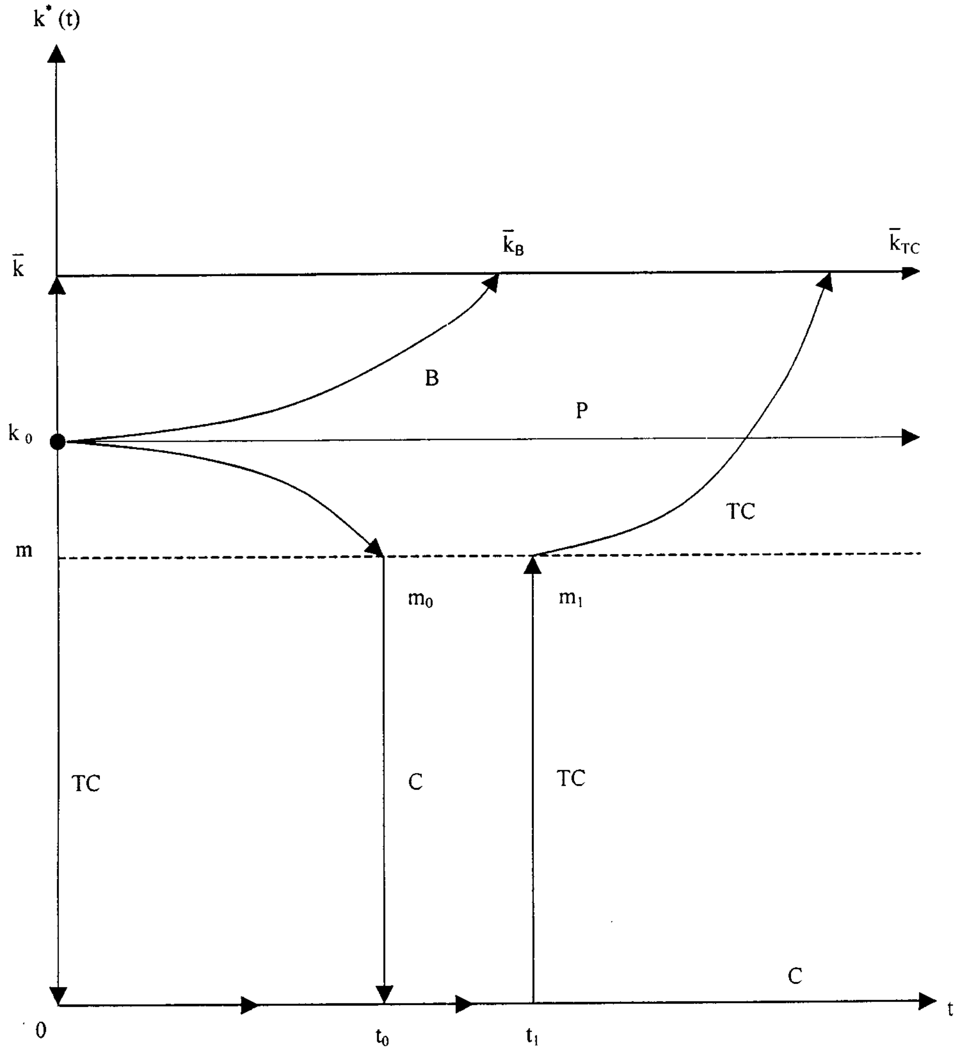

Figure 1. Optimal capital paths for various types of agent.

remain financially constrained, perhaps indefinitely.

Proposition 2: *Some agents choose to be financially constrained forever.*

Proof. Integrate Eq. (8) and substitute into Eq. (5) to obtain N's optimal consumption plan:

$$u_c(c^*(t)) = A \cdot e^{(\rho - r)t}, \qquad (11)$$

where $A > 0$ is a constant. Thus agents with ρ > (<) [=] r desire high (low) [intermediate] initial consumption, followed by a declining [constant] (increasing) consumption path. By (i) of Proposition 1, B's initial consumption must be lower than N's, in order to ensure $a \geq 0$. However, supposing $k_0 > m$, $v(0) = 1$ and $a_0 = 0$ for B, Proposition 1(i) is insufficient to guarantee $c^* \leq f(k_0)$. Thus $i^* < 0$ is possible, and will occur for sufficiently high c^*, i.e., for a sufficiently high ρ relative to r. By Eq. (2), k falls and when $k(t) < m$, $v^*(t) = 0$. The agent closes his business and takes paid employment. For a sufficiently large ρ, desired consumption may continue to exceed income w for all subsequent periods. Thus the agent remains financially constrained forever with $i^*(t) = 0\ \forall t$, i.e., never re-opens his or her business. Of course, if $k_0 < m$, $v(0) = 0$ and $a_0 = 0$ for B, the agent does not start in entrepreneurship and by the same argument will never enter it, for a sufficiently large ρ. □

Figure 1 illustrates the optimal (three-sectioned) capital path for this type of agent, denoted "C" (the path is $k_0 m_0 t_0 t$). Note that this provides a novel explanation for voluntary closure of viable businesses, in contrast with previous theoretical work (e.g., Lucas, 1978; Jovanovic, 1982) that has stressed the importance of lower returns in entrepreneurship relative to paid employment. Entrepreneurs may receive a higher return in entrepreneurship, and yet *still* find it optimal to exit and remain in paid employment thenceforth.

A numerical example may help clarify ideas. Consider an economy with a 10 per cent interest rate, a 5 per cent capital depreciation rate, and a linear production function $f(k) = Ak$ for simplicity:[7] $A = r + \delta = 0.15$. Suppose maximum optimal operating capital is $\bar{k} = 2$ (in units of one hundred thousand pounds): then entrepreneurial income at full capacity is $f(\bar{k}) = 0.30$ in the same units. These are both reasonable figures with regard to the U.K. experience. According to Robson (1997), employee incomes have averaged around 70 per cent of entrepreneurial incomes since the early 1980s. Hence I take $w = 0.21$, so $m = f^{-1}(0.21) = 1.4$. Consider an individual starting with $k_0 = 1.6$ units of physical capital, with a starting income of $f(1.6) = 0.24$. For simplicity, assume $\rho = r$, so the individual has a flat consumption profile, of amount $c^* = 0.06$ in each period. For a capital constrained agent B, $i(t) = f[k(t)] - c$, which in conjunction with Eq. (2) yields a first order linear differential equation $k - (A - \delta)k = -c$, whose solution is $k^*(t) = k_0 e^{(A-\delta)t} + [c^*/(A - \delta)] \cdot [1 - \exp(A - \delta)t\}]$. This can be used to solve for how long it takes the constrained individual to obtain the optimal capital stock of 2, *viz.*: $10 \times \ln 1.4 = 3.36$ years. Obviously, relatively patient individuals will escape the borrowing constraint even sooner than this.

This example also shows that the level of initial assets plays a key role in terms of the timing of business start-ups. An individual starting with zero physical or financial capital would initially be an employee, accumulating wealth according to the equation $\dot{a} = ra + w - c$. The solution to this equation for a flat consumption profile is $a^*(t) = [(w - c)/r] \cdot [\exp\{rt\} - 1]$. Thus the employee takes 6.6 years before they accumulate $m = 1.4$ units of capital, and switch to entrepreneurship. It takes them another 5.6 years to completely overcome the borrowing constraint and operate with capital of $\bar{k} = 2$. A total of 12.2 years to overcome the borrowing constraints shows clearly how important borrowing constraints remain, even taking into account the actions of individuals saving to overcome them.

Returning to the general formulation of the problem, two other types of voluntary constrainedness are also of interest. First, consider an entrepreneur with $\rho = r$, $a_0 = 0$, $m < k_0 < \bar{k}$ and $c^*(t) = f(k_0)$, $\forall t$. This individual, denoted "P" in Figure 1, remains an entrepreneur with $i^* = 0$, but is permanently financially constrained, operating a permanently sub-optimal capital stock k_0.[8] Of course, if $k_0 < m$, all else unchanged, the agent would have remained an employee forever. The second case concerns voluntary closure, followed by subsequent re-opening of a business. To see why this may be optimal, consider a constrained entrepreneur with $\rho < r$, $a_0 = 0$, $m < k_0 < \bar{k}$ and $c^*(t) = f(k_0)$. With a one-off negative shock to current period income, this entrepreneur may optimally have $i^* < 0$ if consumption is not reduced by the full extent of the shock. If this ensures $k^* < m$, then $v^* = 0$ and the entrepreneur closes the business. However, this increases the stock of financial assets by Eq. (3), and with restored post-shock incomes and low initial consumption (see the proof of Proposition 2, above), assets eventually recover to the point where $v^* = 1$ and $k^* = m$: the agent re-opens the business. Physical capital is then accumulated using subsequent savings, as predicted by Proposition 1.[9] Figure 1 illustrates the optimal (five-sectioned) capital path for this type of agent, denoted "TC" (for "temporary closure"). The path is $k_0 0 t_1 m_1 \bar{k}_{TC}$.

Finally, can non-financially constrained agents, N, anticipate future borrowing constraints by adjusting their behaviour in advance? The answer is negative: from N's "myopic" investment rule of $f_{\bar{k}} = r + \delta$ (Proposition 1), only current events determine capital investment decisions. Agents N are indifferent between building up financial or physical assets, since at the optimum both yield a rate of return of r. This reinforces the above finding that it can be optimal for agents to voluntarily succumb to borrowing constraints.

3.2. *Other determinants of entrepreneurship*

As shown above, savings permit accumulation of assets regardless of initial endowments, and so encourage business entry. There are two caveats to this. The first is that some workers may be too poor to save enough, for example if $k(0) = a(0) = 0$ and $a^*(T) < m$, i.e., if $w < mr[e^{rT} - 1]^{-1}$.[10] This may be an important consideration in practice. The second, adumbrated by the numerical example above, is that initial assets are in some cases essential for successful entrepreneurship:

Proposition 3: *Greater entrepreneurial ability increases the likelihood and duration of an agent's borrowing constraint; greater initial financial assets reduce them.*

Proof. Greater ability increases the marginal product of capital at all capital levels, so the equilibrium capital stock $\bar{k}$ is also increasing in ability. However, as $\bar{k}$ increases, so obviously also does the magnitude of any investment needed to attain it. Given an individual's initial asset stock a_0, an increase in i will reduce a by Eq. (3) and increase the likelihood and duration of borrowing constraints. This likelihood is reduced the greater is a_0, since these initial assets can help purchase the necessary investment. □

A corollary of Proposition 3 is that although high ability agents face the steepest borrowing constraints, they also have greater incentives to save to overcome them. This is because their marginal product of capital is highest. In turn, this suggests that in equilibrium, entrepreneurs will have high average ability levels relative to the population as a whole. However, the prediction that the highest ability agents are penalised with the steepest constraints may appear surprising. It is clearly model-specific. For example, it could be overturned in an alternative formulation where ability reduces production costs rather than increases the marginal product of capital.

Proposition 4: *Agents are more likely to become entrepreneurs if they are relatively patient (i.e., have a small inter-temporal discount rate* ρ*) and if the interest rate increases. However, the effect of the employee wage w on entrepreneurship is ambiguous* a priori.

Proof. From Eq. (11), savings are positively related to r and negatively to ρ. Since savings are needed to escape borrowing constraints, the first part of the Proposition follows. For the second part, note that a rise in w has two effects. On one hand it increases the attractiveness of paid employment, so reducing entrepreneurship. On the other, it increases incomes and hence savings, which tends to promote entrepreneurship. □

Two things are noteworthy about Proposition 4. First, the finding that more patient agents are likelier to become entrepreneurs is (to our knowledge) a novel one. It suggests that patience should be added to the list of personal characteristics, such as managerial ability and low risk aversion, which are commonly linked with entrepreneurship. The principle can be illustrated very easily. For example, agent B depicted in Figure 1 has a low discount rate $\rho < r$, and so accumulates assets until $\bar{k}$ is obtained. This compares with agent C of Figure 1, with $\rho > r$, who from Proposition 2 would close a business if they had one and would not enter in the first place if $k_0 < m$. Secondly, the ambiguous effect of the employee wage on occupational choice in a two-sector framework appears to be novel: we know of no other occupational choice model sufficiently rich to share this feature.[11]

3.3. *Welfare implications of borrowing constraints*

Just because individuals may be able to overcome borrowing constraints does not render them welfare-neutral. In fact, there are likely to be substantial welfare losses associated with borrowing constraints. Compared with the unconstrained solution, individuals have to consume less than is optimal and use inefficiently small amounts of capital for the duration of the time they are constrained. The income loss from not having $\bar{k}$ in turn reduces an individual's lifetime wealth compared with the unconstrained case, which means that they must also consume less in all periods, not just while they are constrained. Thus the lifetime utility (welfare) of a given constrained individual is unambiguously less than they would enjoy without borrowing constraints. To the extent that individuals save to overcome the constraint, they reduce the income loss that they would otherwise suffer and so mitigate the welfare loss. Therefore an economy comprised of "patient" agents can be expected to suffer a smaller aggregate welfare loss than one comprised of "impatient" agents.

4. Extending the model

4.1. *Bequests*

The model is easily generalised to allow for bequests. Because physical and financial capital are perfectly convertible, bequests of physical capital can be studied without loss of generality. Suppose agents receive a "warm glow" from bequeathing physical capital when they die.[12] Let $\beta u[k(T)]$ be the undiscounted utility from bequeathing $k(T)$, with $\beta > 0$, $u(0) = 0$, $u_k(\infty) = 0$ and $u_k(0) = 0$.[13] Then the objective function Eq. (1) becomes

$$\begin{aligned}\max\int_0^T u[c(t)]e^{-\rho t}\mathrm{d}t + \beta u[k(T)]e^{-\rho T} \\ = \max\int_0^T \{u[c(t)] - \rho\beta u[k(t)] + \\ \beta u_{k(t)}[i(t) - \delta k(t)]\}e^{-\rho t}\mathrm{d}t. \qquad (12)\end{aligned}$$

Proposition 5: *Without a bequest motive, all entrepreneurs will have closed their businesses by the terminal time. A bequest motive of the type in Eq. (12) reduces the number of such closures.*

The proof of this proposition, given in Parker (1999) and suppressed here for brevity, shows that a bequest motive does not necessarily prevent voluntary closures when entrepreneurs die – it just makes closures less likely. An additional indirect effect of inheritance is of course to increase initial physical assets k_0 for the next generation, which reduces their borrowing constraints (by Proposition 3) and so increases their likelihood of becoming entrepreneurs. This prediction of inter-generational succession of businesses is consistent with the available evidence (see, e.g., Kimhi, 1997).

4.2. *Alternative forms of borrowing constraint*

None of the above results change qualitatively if the constraint $a(t) \geq 0$ is relaxed to become $a(t) \geq \underline{a}$, where $\underline{a} < 0$. It can be easily verified that all of the preceding propositions go through as before, with the exception of the interest rate effect described in Proposition 4 (see footnote 11). The only quantitative difference is that the borrowing constraint is weakened (and becomes increasingly weak as $\underline{a} \to -\infty$). Thus there is lower incidence and duration of borrowing constraints, and more agents choose entrepreneurship. Fewer agents choose to be financially constrained forever, since some agents who chose this under $\underline{a} = 0$ would now find they could optimally borrow enough physical capital to have $k > m$, and so become an entrepreneur. Likewise, the incidence of entrepreneurs operating sub-optimal capital stocks, and of shocks leading to temporary business closures, falls as $\underline{a} \to -\infty$, the latter because debt provides a cushion against temporary downturns. Also, a debt facility weakens the influence of initial assets on the tightness of borrowing constraints.

Two new issues emerge when $\underline{a} < 0$. First, debt servicing requires reductions in consumption and/or investment. To the extent that reduction in investment is used to service the debt, any decline in the number of voluntary closures and the number of businesses operating sub-optimal capital stocks will be mitigated. Second, debts must be repaid in full if rational lenders are to supply funds at all. In a competitive market, $\underline{a}$ will be set at the level that earns lenders normal profits.[14]

Another possibility is that agents can borrow up to a given multiple of their financial assets (if positive), denoted by $\lambda > 0$. This "L-shaped" constraint has been studied by Evans and Jovanovic (1989), amongst others. It can be treated within the above framework in the following way:

$$\underline{a}(t) = \begin{cases} -\lambda a(t) & \text{if} \quad a(t) \geq 0 \\ \underline{a}(t-\Delta) & \text{if} \quad a(t) < 0 \end{cases} \qquad (13)$$

where $\underline{a}(t - \Delta)$ is the most recently agreed debt limit, Δ periods ago. Thus the agent has a *time-variable* debt constraint.[15]

Proposition 6: *A time-variable debt constraint alters none of the qualitative results of the foregoing analysis.*

For the proof, see Parker (1999).

4.3. *Other extensions*

One extension is to relax the partial equilibrium nature of the model, by allowing the number of agents in a sector to reduce average returns there. While this changes none of the Propositions, it does have the implication that the amount of equilibrium occupational switching will increase (see Parker, 1999, for a discussion). However, the effect of greater switching can be cancelled out if the assumption of costless accumulation and decumulation of physical assets is relaxed. Adjustment costs would slow down the speed of adjustment to agents' desired destination, e.g., k for B and N, and zero for C. Likewise, temporary closures would be less likely, since the shock could be over before TC ran down his capital to m and quit entrepreneurship.

This in turn raises the broader question about the model's robustness to risk and uncertainty. Uncertainty about life span can be handled relatively easily. For example, suppose the probability of dying in the interval dt is $\xi \mathrm{d}t + O(\mathrm{d}t)$, $\xi > 0$. Then the problem of optimal choice over an uncertain lifetime is transformed into an equivalent choice problem over an infinite lifetime, where the inter-temporal discount rate is increased by ξ (see Vaughan, 1988). The only implication this has for the results obtained earlier is to augment Proposition 4: now a lower probability of dying as well as greater patience increases the probability of becoming an entrepreneur. Another source of uncertainty is uncertain income streams, e.g., incomes that follow Brownian motions. This problem is difficult to handle when state variables are bounded, although Parker (1996, 1997) studied the entrepreneur's entry decision for the case where no bounds are present. Unsurprisingly, entry into entrepreneurship is more likely, *cet. par.*, the lesser (greater) the income uncertainty in entrepreneurship (paid employment). However, it is not clear *a priori* what effects this type of uncertainty might have on constraint evasion; a full treatment of this problem is reserved for future work.

Finally, the model so far only considered one type of business. What if there are many types of business, where some have lower capital requirements than others? For example, consider a set of heterogeneous production functions, with a corresponding set of switching (entry) points into entrepreneurship, $\{m\}$. Clearly, no production function could dominate all others at all levels of capital, otherwise all entrepreneurs would utilise only one technology. But with a set of production functions embodying different productivity rates at various capital input rates, there is scope for heterogeneous entrepreneurs to select into different technology types depending in part on their willingness to save. For example, relatively impatient individuals could remain in entrepreneurship, but only in technologies requiring relatively low levels of capital. Relatively patient individuals would select into technologies with the greatest capital inputs and returns. The only implication this extension has for the model would be to accentuate the difference between the wealth of patient entrepreneurs relative to impatient entrepreneurs and workers.

5. Conclusions

This paper has shown how agents can overcome borrowing constraints to set up in business and expand their capital to optimal levels. The escape mechanism studied here was life-cycle savings. While alternative forms of constraint avoidance could also be suggested, such as hard work (i.e., inter-temporal labour supply adjustment), the same principles analysed here would continue to apply. It was shown that while some individuals may choose to escape borrowing constraints, others may voluntarily and optimally choose to succumb to them, even at the expense of losing income. The model can also explain temporary and permanent closures of viable businesses. Another key result was that individuals with a low subjective discount rate are more likely to become entrepreneurs. While it is known from standard models that a reduction in the discount rate increases the accumulation of assets, the novelty of this paper is that the incentive to finance a business increases the savings rate of entrepreneurs. That is, entrepreneurs save more because they have a higher return from saving.

Further research is needed, to test the model's rich set of predictions. For example, it would be interesting to utilise psychometric data to test the hypothesised effects of patience on entrepreneurship, and to quantify (perhaps with longitudinal data) the extent to which entrepreneurs do actually overcome borrowing constraints in practice.

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Notes

[1] See Cressy (2000), de Meza and Webb (1987) and de Meza and Southey (1996) for theoretical objections to the borrowing constraint/credit rationing story. These objections are based on, respectively, the non-robustness of Evans and Jovanovic's predicted entry-asset relationship, "overlending" rather than under-lending in the credit market, and excessive entrepreneurial optimism meriting stringent bank lending behaviour.

[2] This set-up is "realistic" in the sense that most debt finance provided by banks in practice is of a fixed term or overdraft type (Bank of England, 1999). I will not treat the case where credit is rationed; see Jaffe and Stiglitz (1990) for a survey of that literature.

[3] It would be possible to relax this assumption without altering the qualitative results that follow, to allow non-decreasing returns in some regions of the production function. However, apart from a numerical example later, which assumes constant returns to scale technology, I will not investigate this possibility further in this paper.

[4] This follows from the assumed absence of entry and exit costs: agents can always costlessly transform physical into financial capital and vice-versa. Implications of relaxing this assumption are considered in Section 4.3.

[5] Of course, this is a partial equilibrium result, since an increased supply of workers into entrepreneurship will in general reduce average returns there. It is also dependent on the assumption of no uncertainty. The implications of relaxing these assumptions are discussed in Section 4.3.

[6] See also Cressy (1996a), who argues that workers with greater human capital receive higher wages and hence have greater assets to invest – implying that human capital is the root cause of business entry or non-entry, not assets *per se*. Xu (1998) uses a simple two period model to show how savings in the first period can increase wealth that can be borrowed against in the second period.

[7] Replacing this with a strictly concave function would lead to an intractable non-linear differential equation in the capital stock (q.v. below), and so is not pursued here.

[8] Strictly, it is necessary that this agent leaves an optimal bequest of physical capital of k_0 for this possibility to obtain. See Section 4.1 for a treatment of capital bequests.

[9] A rigorous proof of the above would require the model to be generalised in order to allow agents to take shocks into account. This would necessitate an optimal stochastic control problem, which introduces complications that go beyond the scope of the present paper (but see Section 4.3).

[10] This follows from the solution to $\dot{a} = ra + w$ being $a(t) = (w/r)\,(e^{rt-1})$.

[11] The effect of the interest rate on entrepreneurial entry outlined in Proposition 4 is somewhat more complicated if $\underline{a} = 0$ is relaxed to become $\underline{a} \leq 0$. If agents are in debt, then a rise in the interest rate will induce more saving, as above, but will also reduce their disposable income (via Eq. (3)). The net effect of the interest rate on the likelihood of entry is therefore ambiguous. However, the prediction about ρ goes through unchanged.

[12] Of course, this is a simplification, since most agents who bequeath businesses do so before they die.

[13] The $u_k(0) = 0$ assumption ensures the realistic outcome that some agents (e.g., non-entrepreneurs) may die without bequeathing any physical capital. The same is not true of the alternative assumption $u_k(0) = \infty$.

[14] Naturally, in practice $\underline{a}$ may vary from person to person. For example, Cressy (1996b) analyses how the structure of borrowing contracts in the U.K. depend on the characteristics of entrepreneurs and their businesses.

[15] It is also possible to generalise Eq. (13) to allow physical capital to also be used as collateral for borrowing. This does alters none of the following results and so is suppressed for brevity.

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